

Electrified Fluctuations in $D1 \perp D3$ and $D1 \perp D5$ Systems

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Abstract

We present the physical phenomenon of the subject discussed in the paper [1]. In that paper we dealt with the fluctuations of funnel solutions of intersecting D1 and D3 branes and the electric field E was considered as very high value causing the results to be non-physical. In the present work, the variation interval of E is to be $[0, \frac{1}{\lambda}]$. Then, we extend the study to discuss the overall transverse fluctuations of electrified funnel solutions of $D1 \perp D5$ system in the flat background. The boundary conditions are found to be Neumann boundary conditions.

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1 Introduction

D-branes are extended objects, topological defects in string theory on which string endpoints can live. These objects have brought about significant advances in string theory. Various brane configurations have attracted much attention over recent years and several papers have been devoted to the study of the relationship between D-branes with different dimensions [2, 3, 4]. Much of the progress has come about by directly studying the low energy dynamics of the D-branes world volume which is known to be governed by the Born-Infeld (BI) action [5, 6, 7, 8]. This world volume theory living on D-branes has many fascinating features. Among these there is the possibility for Dp-branes, through an appropriate excitation of fields, to morph into objects resembling Dp-branes of lower or higher dimensionality. The dynamics of their solutions (bion spike [6, 9, 10, 11] and fuzzy funnel [12]) were studied by considering linearized fluctuations around the static solutions [13, 14]. In the present paper, we go on to study the fluctuations of the fuzzy funnel at the presence of electric field and to limit ourselves to treat the physical phenomenon we consider the electric field in the interval $[0, \frac{1}{\lambda}]$.

The primary goal of this work is to determine the boundary conditions of $D1 \perp D3$ and $D1 \perp D5$ branes in solving the equations of motion of the fuzzy funnel's fluctuations and discussing the associated potentials. We remark that the variation of the potential V in terms of electric field E and the spatial coordinate σ in non-zero modes for both overall and relative transverse fluctuations in $D1 \perp D3$ system and in zero mode of overall transverse fluctuations in $D1 \perp D5$ system shows a singularity at some stage of σ . This is more clearly seen at the presence of electric field which leads to separate the system into two regions; small and large σ depending on electric field. This implies that these intersecting branes obey Neumann boundary conditions and the end of open string can move freely on the brane. Consequently, the idea that the end of a string ending on a Dp-brane can be seen as an electrically charged particle is supported by the present result. The obtained result in $D1 \perp D3$ system is agree with its dual discussed in [11] considering Born-Infeld action dealing with the fluctuation of the bion spike in $D3 \perp D1$ branes case. The dual case of $D1 \perp D5$ branes is not yet discussed.

This work is organized as follows: In section 2, we start by a brief review on $D1 \perp D3$ and $D1 \perp D5$ branes in dyonic case by using the abelian and non-abelian BI actions [15, 16, 2]. In section 3, we study the electrified fluctuations of fuzzy funnel solutions corresponding to D3 and D5 branes. In the first subsection, we review the zero and non-zero modes of the overall transverse electrified fluctuations of fuzzy funnel solutions given in [14]. Then we discuss the zero and non-zero modes of the relative transverse fluctuations in the second subsection. In the third subsection, we treat the electrified fluctuations of the fuzzy funnel in $D1 \perp D5$ system. We study the solutions of the linearized equations of motion of the overall transverse fluctuations in zero mode and we discuss its associated potential at the extremities of σ . The conclusion is presented in section 4.

2 Intersecting Branes

In this section, we review in brief the intersection of D1 branes with D3 and D5 branes. We focus our study on the presence of electric field and its influence on the potentials and the fluctuations of the fuzzy funnels.

2.1 Electrified D1⊥D3 System

We start by giving in brief the known solutions of intersecting D1-D3 branes. From the point of view of D3 brane description the configuration is described by a monopole on its world volume. We use the abelian BI action and one excited transverse scalar in dyonic case to give the bion solution [2, 17]. The system is described by the following action

$$\begin{aligned}
S = \int dt L &= -T_3 \int d^4\sigma \sqrt{-\det(\eta_{ab} + \lambda^2 \partial_a \phi^i \partial_b \phi^i + \lambda F_{ab})} \\
&= -T_3 \int d^4\sigma \left[1 + \lambda^2 \left(|\nabla\phi|^2 + \vec{B}^2 + \vec{E}^2 \right) \right. \\
&\quad \left. + \lambda^4 \left((\vec{B} \cdot \nabla\phi)^2 + (\vec{E} \cdot \vec{B})^2 + |\vec{E} \wedge \nabla\phi|^2 \right) \right]^{\frac{1}{2}}
\end{aligned} \tag{1}$$

in which F_{ab} ($a, b = 0, \dots, 3$) is the field strength and the electric field is denoted as $F_{0a} = E_a$. σ^a denote the world volume coordinates while ϕ^i ($i = 4, \dots, 9$) are the scalars describing transverse fluctuations of the brane and $\lambda = 2\pi\ell_s^2$ with ℓ_s is the string length. In our case we excite just one scalar so $\phi^i = \phi^9 \equiv \phi$. Following the same process used in the reference [2] by considering static gauge, we look for the lowest energy of the system. Accordingly to (1) the energy of dyonic system is given as

$$\begin{aligned}
\Xi &= T_3 \int d^3\sigma \left[\lambda^2 |\nabla\phi + \vec{B} + \vec{E}|^2 + (1 - \lambda^2 \nabla\phi \cdot \vec{B})^2 - 2\lambda^2 \vec{E} \cdot (\vec{B} + \nabla\phi) \right. \\
&\quad \left. + \lambda^4 \left((\vec{E} \cdot \vec{B})^2 + |\vec{E} \wedge \nabla\phi|^2 \right) \right]^{1/2}.
\end{aligned} \tag{2}$$

Then if we require $\nabla\phi + \vec{B} + \vec{E} = 0$, Ξ reduces to

$$\begin{aligned}
\Xi_0 &= T_3 \int d^3\sigma \left[(1 - \lambda^2 (\nabla\phi \cdot \vec{B}))^2 + 2\lambda^2 \vec{E} \cdot \vec{E} \right. \\
&\quad \left. + \lambda^4 ((\vec{E} \cdot \vec{B})^2 + |\vec{E} \wedge \nabla\phi|^2) \right]^{1/2}
\end{aligned} \tag{3}$$

as minimum energy. By using the Bianchi identity $\nabla \cdot \vec{B} = 0$ and the fact that the gauge field is static, the bion solution is then

$$\phi = \frac{N_m + N_e}{2r}, \tag{4}$$

with N_m is magnetic charge and N_e electric charge.

Now we consider the dual description of the D1⊥D3 branes from D1 branes point of view. To get D3-branes from D-strings, we use the non-abelian BI action. The natural definition of this action suggested in [18] is based on replacing the field strength in the BI action by a non-abelian field strength and adding the symmetrized trace $STr(\dots)$ in front of the $\sqrt{\det}$ action. The precise prescription proposed in [18] was that inside the trace one takes a symmetrized average over orderings of the field strength. We refer the reader to [18] for more details on this action.

The non-abelian BI action describing D-string opening up into a D3-brane is given by

$$S = -T_1 \int d^2\sigma STr \left[-\det(\eta_{ab} + \lambda^2 \partial_a \phi^i Q_{ij}^{-1} \partial_b \phi^j) \det Q^{ij} \right]^{\frac{1}{2}} \tag{5}$$

where $Q_{ij} = \delta_{ij} + i\lambda[\phi_i, \phi_j]$. Expanding this action to leading order in λ yields the usual non-abelian scalar action

$$S \cong -T_1 \int d^2\sigma \left[N + \lambda^2 \text{Tr}(\partial_a \phi^i + \frac{1}{2}[\phi_i, \phi_j][\phi_j, \phi_i]) + \dots \right]^{\frac{1}{2}}.$$

We deal with the leading order in N when we expand the symmetrized trace and we consider large N limit. The solutions of the equation of motion of the scalar fields ϕ_i , $i = 1, 2, 3$ represent the D-string expanding into a D3-brane analogous to the bion solution of the D3-brane theory [6, 7]. The solutions are

$$\phi_i = \pm \frac{\alpha_i}{2\sigma}, \quad [\alpha_i, \alpha_j] = 2i\epsilon^{ijk}\alpha_k,$$

with the corresponding geometry is a long funnel where the cross-section at fixed σ has the topology of a fuzzy two-sphere.

The dyonic case is presented by considering (N, N_f) -strings. We introduce a background U(1) electric field on the N D-strings, corresponding to N_f fundamental strings dissolved on the world sheet [10]. The theory is described by the action

$$S = -T_1 \int d^2\sigma \text{STr} \left[-\det(\eta_{ab} + \lambda^2 \partial_a \phi^i Q_{ij}^{-1} \partial_b \phi^j + \lambda F_{ab}) \det Q^{ij} \right]^{\frac{1}{2}}. \quad (6)$$

The action can be rewritten as

$$S = -T_1 \int d^2\sigma \text{STr} \left[-\det \begin{pmatrix} \eta_{ab} + \lambda F_{ab} & \lambda \partial_a \phi^j \\ -\lambda \partial_b \phi^i & Q^{ij} \end{pmatrix} \right]^{\frac{1}{2}}. \quad (7)$$

By computing the determinant, the action becomes

$$S = -T_1 \int d^2\sigma \text{STr} \left[(1 - \lambda^2 E^2 + \alpha_i \alpha_i \hat{R}^2)(1 + 4\lambda^2 \alpha_j \alpha_j \hat{R}^4) \right]^{\frac{1}{2}}, \quad (8)$$

in which we replaced the field strength $F_{\tau\sigma}$ by $E I_N$ (I_N is $N \times N$ -matrix) and the following ansatz were inserted

$$\phi_i = \hat{R} \alpha_i. \quad (9)$$

Hence, we get the funnel solution for dyonic string by solving the equation of variation of \hat{R} as follows

$$\phi_i = \frac{\alpha_i}{2\sigma \sqrt{1 - \lambda^2 E^2}}. \quad (10)$$

2.2 Electrified D1⊥D5 Branes

The fuzzy funnel configuration in which the D-strings expand into orthogonal D5-branes shares many common features with the D3-brane funnel. The action describing the static configurations involving five nontrivial scalars is

$$\begin{aligned} S = & -T_1 \int d^2\sigma \text{STr} \left[1 + \lambda^2 (\partial_\sigma \Phi_i)^2 + 2\lambda^2 \Phi_{ij} \Phi_{ji} + 2\lambda^4 (\Phi_{ij} \Phi_{ji})^2 - 4\lambda^4 \Phi_{ij} \Phi_{jk} \Phi_{kl} \Phi_{li} \right. \\ & \left. + 2\lambda^4 (\partial_\sigma \Phi_{ij})^2 \Phi_{jk} \Phi_{kj} - 4\lambda^4 \partial_\sigma \Phi_i \Phi_{ij} \Phi_{jk} \partial_\sigma \Phi_k + \frac{\lambda^6}{4} (\epsilon_{ijklm} \partial_\sigma \Phi_i \Phi_{jk} \Phi_{lm})^2 \right]^{\frac{1}{2}}, \end{aligned} \quad (11)$$

where $\Phi_{ij} \equiv \frac{1}{2}[\Phi_i, \Phi_j]$ and the funnel solution is given by suggesting the following ansatz

$$\Phi_i(\sigma) = \mp \hat{R}(\sigma) G_i, \quad (12)$$

$i = 1, \dots, 5$, where $\hat{R}(\sigma)$ is the (positive) radial profile and G_i are the matrices constructed in [20]. We note that G_i are given by the totally symmetric n -fold tensor product of 4×4 gamma matrices, and that the dimension of the matrices is related to the integer n by $N = \frac{(n+1)(n+2)(n+3)}{6}$. The Funnel solution (12) has the following physical radius

$$R(\sigma) = \frac{\lambda}{N} \sqrt{\text{Tr}(\Phi_i)^2} = \sqrt{c} \lambda \hat{R}(\sigma), \quad (13)$$

with c is the Casimir associated with the G_i matrices, given by $c = n(n+4)$ and the resulting action for the radial profile $R(\sigma)$ is

$$S = -NT_1 \int d^2\sigma \sqrt{1 + (R')^2} \left(1 + 4 \frac{R^4}{c\lambda^2}\right). \quad (14)$$

We note that this result only captures the leading large N contribution at each order in the expansion of the square root.

To extend the discussion to dyonic strings we consider (N, N_f) -strings. Thus, the electric field is on and the system dyonic is described by the action

$$S = -T_1 \int d^2\sigma \text{STr} \left[-\det(\eta_{ab} + \lambda^2 \partial_a \Phi^i Q_{ij}^{-1} \partial_b \Phi^j + \lambda F_{ab}) \det Q^{ij} \right]^{\frac{1}{2}}. \quad (15)$$

The action can be rewritten as

$$S = -T_1 \int d^2\sigma \text{STr} \left[-\det \begin{pmatrix} \eta_{ab} + \lambda F_{ab} & \lambda \partial_a \Phi^j \\ -\lambda \partial_b \Phi^i & Q^{ij} \end{pmatrix} \right]^{\frac{1}{2}}, \quad (15')$$

with $Q_{ij} = \delta_{ij} + i\lambda[\Phi_i, \Phi_j]$ and $i, j = 1, \dots, 5$, $a, b = \tau, \sigma$. We insert the ansatz (12) and $F_{\tau\sigma} = EI_N$ (I_N is $N \times N$ -matrix) in the action (15). Then we compute the determinant and we obtain

$$S = -NT_1 \int d^2\sigma \sqrt{1 - \lambda^2 E^2 + (R')^2} \left(1 + 4 \frac{R^4}{c\lambda^2}\right). \quad (15'')$$

The funnel solution is

$$\Phi_i(\sigma) = \mp \frac{R(\sigma)}{\lambda \sqrt{c}} G_i. \quad (16)$$

From (15'') We can derive the lowest energy

$$\begin{aligned} E &= -NT_1 \int d\sigma \sqrt{\left(\sqrt{1 - \lambda^2 E^2} \pm R' \sqrt{\frac{8R^4}{c\lambda^2} + \frac{16R^8}{c^2\lambda^4}}\right)^2 + \left(R' \mp \sqrt{1 - \lambda^2 E^2} \sqrt{\frac{8R^4}{c\lambda^2} + \frac{16R^8}{c^2\lambda^4}}\right)^2} \\ &\geq -NT_1 \int d\sigma \left(\sqrt{1 - \lambda^2 E^2} \pm R' \sqrt{\frac{8R^4}{c\lambda^2} + \frac{16R^8}{c^2\lambda^4}}\right). \end{aligned}$$

This is obtained when

$$R' = \mp \sqrt{1 - \lambda^2 E^2} \sqrt{\frac{8R^4}{c\lambda^2} + \frac{16R^8}{c^2\lambda^4}}.$$

This equation can be explicitly solved in terms of elliptic functions. For small R , the R^4 term under the square root dominates, and we find the funnel solution. Then the physical radius of the fuzzy funnel solution (16) is found to be

$$R \approx \frac{\lambda\sqrt{c}}{2\sqrt{2}\sqrt{1-\lambda^2 E^2}\sigma}. \quad (17)$$

In the next sections, we give an examination of the propagation of the fluctuations on the fuzzy funnel. The setup is similar to both $D1\perp D5$ and $D1\perp D3$ systems. We notice that there are two basic types of funnel's fluctuations, the overall transverse ones in the directions perpendicular to both the Dp-brane ($p=3,5$) and the string (i.e., $X^{p+1,\dots,8}$), and the relative transverse ones which are transverse to the string, but parallel to the Dp-brane world volume (i.e., along $X^{1,\dots,p}$).

3 Dyonic Funnel's Fluctuations

In this section, we treat the dynamics of the funnel solutions. We solve the linearized equations of motion for small and time-dependent fluctuations of the transverse scalars around the exact background in dyonic case.

3.1 Overall Transverse Fluctuations in $D1\perp D3$ System

3.1.1 Zero Mode

We deal with the fluctuations of the funnel (10) discussed in the previous section. By plugging into the full (N, N_f) -string action (6,7) the "overall transverse" $\delta\phi^m(\sigma, t) = f^m(\sigma, t)I_N$, $m = 4, \dots, 8$ which is the simplest type of fluctuation with I_N the identity matrix, together with the funnel solution, we get

$$\begin{aligned} S &= -T_1 \int d^2\sigma ST r \left[(1 + \lambda E) \left(1 + \frac{\lambda^2 \alpha^i \alpha^i}{4(1 - \lambda^2 E^2)^2 \sigma^4} \right) \right. \\ &\quad \left. \left(\left(1 + \frac{\lambda^2 \alpha^i \alpha^i}{4(1 - \lambda^2 E^2)^2 \sigma^4} \right) (1 - (1 - \lambda E) \lambda^2 (\partial_t \delta\phi^m)^2) + \lambda^2 (\partial_\sigma \delta\phi^m)^2 \right) \right]^{\frac{1}{2}} \\ &\approx -NT_1 \int d^2\sigma H \left[(1 + \lambda E) - (1 - \lambda^2 E^2) \frac{\lambda^2}{2} (\dot{f}^m)^2 + \frac{(1 + \lambda E)\lambda^2}{2H} (\partial_\sigma f^m)^2 + \dots \right] \end{aligned} \quad (18)$$

where

$$H = 1 + \frac{\lambda^2 C}{4(1 - \lambda^2 E^2)^2 \sigma^4}$$

and $C = Tr \alpha^i \alpha^i$. For the irreducible $N \times N$ representation we have $C = N^2 - 1$. In the last line we have only kept the terms quadratic in the fluctuations as this is sufficient to determine the linearized equations of motion

$$\left((1 - \lambda E) \left(1 + \lambda^2 \frac{N^2 - 1}{4(1 - \lambda^2 E^2)^2 \sigma^4} \right) \partial_t^2 - \partial_\sigma^2 \right) f^m = 0. \quad (19)$$

In the overall case, all the points of the fuzzy funnel move or fluctuate in the same direction of the dyonic string by an equal distance δx^m . Thus, the fluctuations f^m could be rewritten as follows

$$f^m(\sigma, t) = \Phi(\sigma) e^{-i\omega t} \delta x^m, \quad (20)$$

where Φ is a function of the spatial coordinate. With this ansatz the equation of motion (19) becomes

$$\left((1 - \lambda E)\left(1 + \lambda^2 \frac{N^2 - 1}{4(1 - \lambda^2 E^2)^2 \sigma^4}\right)w^2 + \partial_\sigma^2\right)\Phi(\sigma) = 0. \quad (21)$$

Then, the problem is reduced to finding the solution of a single scalar equation.

In this work, we consider the physical phenomenon which is defined by the fact that the electric field E is in the interval $[0, \frac{1}{\lambda}]$ (contrary to what was treated in [1], such that E was tending to ∞).

The equation (21) is an analog one-dimensional Schrödinger equation. Let's rewrite it as

$$\left(\frac{1}{w^2(1 - \lambda E)}\partial_\sigma^2 + 1 + \frac{\lambda^2 N^2}{4(1 - \lambda^2 E^2)^2 \sigma^4}\right)\Phi(\sigma) = 0, \quad (22)$$

for large N . If we suggest

$$\tilde{\sigma} = w\sqrt{1 - \lambda E}\sigma, \quad (23)$$

the equation (22) becomes

$$\left(\partial_{\tilde{\sigma}}^2 + 1 + \frac{\kappa^2}{\tilde{\sigma}^4}\right)\Phi(\tilde{\sigma}) = 0, \quad (24)$$

with the potential is

$$V(\tilde{\sigma}) = \frac{\kappa^2}{\tilde{\sigma}^4}, \quad (25)$$

and

$$\kappa = \frac{\lambda N w^2}{2(1 + \lambda E)}. \quad (26)$$

The equation (24) is a Schrödinger equation for an attractive singular potential $\propto \tilde{\sigma}^{-4}$ and depends on the single coupling parameter κ with constant positive Schrödinger energy. The solution is then known by making the following coordinate change

$$\chi(\tilde{\sigma}) = \int_{\sqrt{\kappa}}^{\tilde{\sigma}} dy \sqrt{1 + \frac{\kappa^2}{y^4}}, \quad (27)$$

and

$$\Phi = \left(1 + \frac{\kappa^2}{\tilde{\sigma}^4}\right)^{-\frac{1}{4}} \tilde{\Phi}. \quad (28)$$

Thus, the equation (24) becomes

$$\left(-\partial_\chi^2 + V(\chi)\right)\tilde{\Phi} = 0, \quad (29)$$

with

$$V(\chi) = \frac{5\kappa^2}{(\tilde{\sigma}^2 + \frac{\kappa^2}{\tilde{\sigma}^2})^3}. \quad (30)$$

Accordingly to the variation of this potential (Fig.1), the system looks like separated into two regions depending on σ . In small σ region V is close to 0 with a constant value for all E . In large σ region, specially when σ reaches 0.7, V increases too fast as we jump to a new region and gets a maximum value when $E \approx 0.5$.

Then, the fluctuation is found to be

$$\Phi = (1 + \frac{\kappa^2}{\tilde{\sigma}^4})^{-\frac{1}{4}} e^{\pm i\chi(\tilde{\sigma})}. \quad (31)$$

This fluctuation has the following limits; at large σ , $\Phi \sim e^{\pm i\chi(\tilde{\sigma})}$ and if σ is small $\Phi = \frac{\tilde{\sigma}}{\sqrt{\kappa}} e^{\pm i\chi(\tilde{\sigma})}$. These are the asymptotic wave function in the regions $\chi \rightarrow \pm\infty$, while around $\chi \sim 0$; i.e. $\tilde{\sigma} \sim \sqrt{\kappa}$, $\Phi \sim 2^{-\frac{1}{4}}$. Also we find that Φ has different expressions in small and large σ regions.

3.1.2 Non-Zero Modes

The fluctuations discussed above could be called the zero mode $\ell = 0$ and for non-zero modes $\ell \geq 0$, the fluctuations are $\delta\phi^m(\sigma, t) = \sum_{\ell=0}^{N-1} \psi_{i_1 \dots i_\ell}^m \alpha^{i_1} \dots \alpha^{i_\ell}$ with $\psi_{i_1 \dots i_\ell}^m$ are completely symmetric and traceless in the lower indices.

The action describing this system is

$$\begin{aligned} S \approx & -NT_1 \int d^2\sigma \left[(1 + \lambda E)H - (1 - \lambda^2 E^2)H \frac{\lambda^2}{2} (\partial_t \delta\phi^m)^2 \right. \\ & + \frac{(1 + \lambda E)\lambda^2}{2H} (\partial_\sigma \delta\phi^m)^2 - (1 - \lambda^2 E^2) \frac{\lambda^2}{2} [\phi^i, \delta\phi^m]^2 \\ & \left. - \frac{\lambda^4}{12} [\partial_\sigma \phi^i, \partial_t \delta\phi^m]^2 + \dots \right] \end{aligned} \quad (32)$$

Now the linearized equations of motion are

$$\left[(1 + \lambda E)H \partial_t^2 - \partial_\sigma^2 \right] \delta\phi^m + (1 - \lambda^2 E^2) [\phi^i, [\phi^i, \delta\phi^m]] - \frac{\lambda^2}{6} [\partial_\sigma \phi^i, [\partial_\sigma \phi^i, \partial_t^2 \delta\phi^m]] = 0, \quad (33)$$

with $H = 1 + \lambda^2 \frac{N^2 - 1}{4(1 - \lambda^2 E^2)^2 \sigma^4}$. Since the background solution is $\phi^i \propto \alpha^i$ and we have $[\alpha^i, \alpha^j] = 2i\epsilon_{ijk}\alpha^k$, we get

$$\begin{aligned} [\alpha^i, [\alpha^i, \delta\phi^m]] &= \sum_{\ell < N} \psi_{i_1 \dots i_\ell}^m [\alpha^i, [\alpha^i, \alpha^{i_1} \dots \alpha^{i_\ell}]] \\ &= \sum_{\ell < N} 4\ell(\ell + 1) \psi_{i_1 \dots i_\ell}^m \alpha^{i_1} \dots \alpha^{i_\ell} \end{aligned} \quad (34)$$

To obtain a specific spherical harmonic on 2-sphere, we have

$$[\phi^i, [\phi^i, \delta\phi_\ell^m]] = \frac{\ell(\ell + 1)}{(1 - \lambda^2 E^2)\sigma^2} \delta\phi_\ell^m, \quad [\partial_\sigma \phi^i, [\partial_\sigma \phi^i, \partial_t^2 \delta\phi_\ell^m]] = \frac{\ell(\ell + 1)}{(1 - \lambda^2 E^2)^2 \sigma^4} \partial_t^2 \delta\phi_\ell^m. \quad (35)$$

Then for each mode the equations of motion are

$$\left[\left((1 + \lambda E) \left(1 + \lambda^2 \frac{N^2 - 1}{4(1 - \lambda^2 E^2)^2 \sigma^4} \right) - \frac{\lambda^2 \ell(\ell + 1)}{6(1 - \lambda^2 E^2)^2 \sigma^4} \right) \partial_t^2 - \partial_\sigma^2 + \frac{\ell(\ell + 1)}{\sigma^2} \right] \delta\phi_\ell^m = 0. \quad (36)$$

The solution of the equation of motion can be found by taking the following proposal. Let's consider $\phi_\ell^m = f_\ell^m(\sigma) e^{-i\omega t} \delta x^m$ in direction m with $f_\ell^m(\sigma)$ is some function of σ for each mode ℓ .

The last equation can be rewritten as

$$\left[-\partial_\sigma^2 + V(\sigma) \right] f_\ell^m(\sigma) = w^2(1 + \lambda E) f_\ell^m(\sigma), \quad (37)$$

with

$$V(\sigma) = -w^2 \left((1 + \lambda E) \frac{\lambda^2 N^2}{4(1 - \lambda^2 E^2)^2 \sigma^4} - \frac{\lambda^2 \ell(\ell + 1)}{6(1 - \lambda^2 E^2)^2 \sigma^4} \right) + \frac{\ell(\ell + 1)}{\sigma^2}.$$

In small σ region, this potential is reduced to

$$V(\sigma) = \frac{-w^2 \lambda^2}{(1 - \lambda^2 E^2)^2 \sigma^4} \left(\frac{(1 + \lambda E) N^2}{4} - \frac{\ell(\ell + 1)}{6} \right).$$

This potential (Fig.2(a)) is close to 0 for almost of σ and E until that $E \approx 0.87$ we remark that V changes at $\sigma \approx 0.04$ and then goes up too fast to be close to 0 again for the other values of σ .

In small σ limit, we reduce the equation (37) to the following form

$$\left[w^2 \left((1 + \lambda E) \left(1 + \lambda^2 \frac{N^2 - 1}{4(1 - \lambda^2 E^2)^2 \sigma^4} \right) - \frac{\lambda^2 \ell(\ell + 1)}{6(1 - \lambda^2 E^2)^2 \sigma^4} \right) + \partial_\sigma^2 \right] f_\ell^m(\sigma) = 0. \quad (38)$$

and again as

$$\left[1 + \frac{1}{(1 - \lambda^2 E^2)^2 \sigma^4} \left(\lambda^2 \frac{N^2 - 1}{4} - \frac{\lambda^2 \ell(\ell + 1)}{6(1 + \lambda E)} \right) + \frac{1}{w^2(1 + \lambda E)} \partial_\sigma^2 \right] f_\ell^m(\sigma) = 0. \quad (39)$$

We define new coordinate $\tilde{\sigma} = w\sqrt{1 + \lambda E}\sigma$ and the latter equation becomes

$$\left[\partial_{\tilde{\sigma}}^2 + 1 + \frac{\kappa^2}{\tilde{\sigma}^4} \right] f_\ell^m(\sigma) = 0, \quad (40)$$

where

$$\kappa^2 = \frac{w^2(1 + \lambda E)}{(1 - \lambda^2 E^2)^2} \left(\lambda^2 \frac{N^2 - 1}{4} - \frac{\lambda^2 \ell(\ell + 1)}{6(1 + \lambda E)} \right)^{\frac{1}{2}}$$

such that

$$N > \sqrt{\frac{2\ell(\ell + 1)}{3(1 + \lambda E)}} + 1.$$

By following the same setup of zero mode, we get the solution by using the steps (27-31) with new κ . Since we considered small σ we get

$$V(\chi) = \frac{5\tilde{\sigma}^6}{\kappa^4}, \quad (41)$$

and the fluctuation is found to be

$$f_\ell^m = \frac{\tilde{\sigma}}{\sqrt{\kappa}} e^{\pm i\chi(\tilde{\sigma})}. \quad (42)$$

in small σ region.

Now, let's check the case of large σ . In this case, the equation of motion (37) of the fluctuation can be rewritten in the following form

$$\left[-\partial_\sigma^2 + V(\sigma) \right] f_\ell^m(\sigma) = w^2(1 + \lambda E) f_\ell^m(\sigma), \quad (43)$$

with

$$V(\sigma) = \frac{\ell(\ell+1)}{\sigma^2}.$$

We remark that, in large σ limit (Fig.2(b)), the potential V is independent of E and going down as σ is going up. The figures 2(a) and 2(b) show that the system in non-zero modes is separated to two totally different regions and the main remark is that the potential gets a singularity at some level of σ which is considered the intersection of small and large σ regions. In our calculations we took small σ from zero until the half of the unit of $\lambda = 1$ and the large σ region from 0.5 until 1 with $w = 1$, $l = 1$ and $N = 10$.

The f_ℓ^m is now a Sturm-Liouville eigenvalue problem. The fluctuation is found to be

$$\begin{aligned} f_\ell^m(\sigma) &= \alpha \sqrt{\sigma} \text{Bessel} J\left(\frac{1}{2}\sqrt{1+4\ell(\ell+1)}, w\sigma\sqrt{1+\lambda E}\right) \\ &+ \beta \sqrt{\sigma} \text{Bessel} Y\left(\frac{1}{2}\sqrt{1+4\ell(\ell+1)}, w\sigma\sqrt{1+\lambda E}\right), \end{aligned} \quad (44)$$

with α, β are constants. Again, it's clear that the fluctuation solution in this case is totally different from the one gotten in small σ limit (43) supporting the idea that the system is divided to two regions. In the following, we continue the study of D1 \perp D3 branes by dealing with the relative transverse fluctuations.

3.2 Relative Transverse Fluctuations in D1 \perp D3 System

3.2.1 Zero Mode

In this subsection, we consider the "relative transverse" fluctuations $\delta\phi^i(\sigma, t) = f^i(\sigma, t)I_N$, $i = 1, 2, 3$, and the action describing the system has the expression

$$S = -T_1 \int d^2\sigma \text{Str} \left[-\det \begin{pmatrix} \eta_{ab} + \lambda F_{ab} & \lambda \partial_a(\phi^j + \delta\phi^j) \\ -\lambda \partial_b(\phi^i + \delta\phi^i) & Q_*^{ij} \end{pmatrix} \right]^{\frac{1}{2}}, \quad (45)$$

with

$$Q_*^{ij} = Q^{ij} + i\lambda([\phi_i, \delta\phi_j] + [\delta\phi_i, \phi_j] + [\delta\phi_i, \delta\phi_j]).$$

As done above, we keep only the terms quadratic in the fluctuations and the action becomes

$$S \approx -NT_1 \int d^2\sigma \left[(1 - \lambda^2 E^2)H - (1 - \lambda E)\frac{\lambda^2}{2}(\dot{f}^i)^2 + \frac{(1 + \lambda E)\lambda^2}{2H}(\partial_\sigma f^i)^2 + \dots \right], \quad (46)$$

with $H = (1 + \lambda^2 \frac{N^2 - 1}{4(1 - \lambda^2 E^2)^2 \sigma^4})$.

Then we define the relative transverse fluctuation as $f^i = \Phi^i(\sigma)e^{-i\omega t}\delta x^i$ in the direction of x^i , with Φ is a function of σ , and the equations of motion of the fluctuations are found to be

$$\left(-\partial_\sigma^2 - \frac{w^2 \lambda^2 (1 - \lambda E)(N^2 - 1)}{4(1 + \lambda E)(1 - \lambda^2 E^2)^2 \sigma^4} \right) \Phi^i = w^2 \frac{1 - \lambda E}{1 + \lambda E} \Phi^i, \quad (47)$$

where the potential is

$$V(\sigma) = -\frac{w^2 \lambda^2 (1 - \lambda E)(N^2 - 1)}{4(1 + \lambda E)(1 - \lambda^2 E^2)^2 \sigma^4}.$$

We remark that the presence of E is quickly increasing the potential from $-\infty$ to zero. Then, when E is close to the inverse of λ the potential is close to zero for all σ ;

- $E \sim 0$, $V(\sigma) \sim -\lambda^2 \frac{N^2-1}{4\sigma^4} w^2$
- $E \sim \frac{1}{\lambda}$, $V(\sigma) \sim -\frac{1-\lambda E}{2} \lambda^2 \frac{N^2-1}{4(1-\lambda^2 E^2)^2 \sigma^4} w^2$.

This case is seen as a zero mode of what is following so we will focus on its general case known as non-zero modes.

3.2.2 Non-Zero Modes

Let's give the equation of motion of relative transverse fluctuations of non-zero ℓ modes with (N, N_f) -strings intersecting D3-branes. The fluctuation is given by $\delta\phi^i(\sigma, t) = \sum_{\ell=1}^{N-1} \psi_{i_1 \dots i_\ell}^i \alpha^{i_1} \dots \alpha^{i_\ell}$ with $\psi_{i_1 \dots i_\ell}^i$ are completely symmetric and traceless in the lower indices. The action describing this system is

$$\begin{aligned} S \approx & -NT_1 \int d^2\sigma \left[(1 - \lambda^2 E^2)H - (1 - \lambda E)H \frac{\lambda^2}{2} (\partial_t \delta\phi^i)^2 \right. \\ & + \frac{(1+\lambda E)\lambda^2}{2H} (\partial_\sigma \delta\phi^i)^2 - (1 - \lambda E) \frac{\lambda^2}{2} [\phi^i, \delta\phi^i]^2 \\ & \left. - \frac{\lambda^4}{12} [\partial_\sigma \phi^i, \partial_t \delta\phi^i]^2 + \dots \right]. \end{aligned} \quad (48)$$

The equation of motion for relative transverse fluctuations in non-zero modes is

$$\left[\frac{1 - \lambda E}{1 + \lambda E} H \partial_t^2 - \partial_\sigma^2 \right] \delta\phi^i + (1 - \lambda E) [\phi^i, [\phi^i, \delta\phi^i]] - \frac{\lambda^2}{6} [\partial_\sigma \phi^i, [\partial_\sigma \phi^i, \partial_t^2 \delta\phi^i]] = 0. \quad (49)$$

By the same way followed in overall case the equation of motion for each mode ℓ is found to be

$$\left[-\partial_\sigma^2 + \left(\frac{1 - \lambda E}{1 + \lambda E} (1 + \lambda^2 \frac{N^2 - 1}{4(1 - \lambda^2 E^2)^2 \sigma^4}) - \frac{\lambda^2 \ell(\ell + 1)}{6(1 - \lambda^2 E^2)^2 \sigma^4} \right) \partial_t^2 + \frac{\ell(\ell + 1)}{(1 + \lambda E)\sigma^2} \right] \delta\phi_\ell^i = 0. \quad (50)$$

We write $\delta\phi_\ell^i = f_\ell^i e^{-i\omega t} \delta x^i$ in the direction of x^i , then the equation (51) becomes

$$\left[-\partial_\sigma^2 - \left(\frac{1 - \lambda E}{1 + \lambda E} (1 + \lambda^2 \frac{N^2 - 1}{4(1 - \lambda^2 E^2)^2 \sigma^4}) - \frac{\lambda^2 \ell(\ell + 1)}{6(1 - \lambda^2 E^2)^2 \sigma^4} \right) w^2 + \frac{\ell(\ell + 1)}{(1 + \lambda E)\sigma^2} \right] f_\ell^i = 0. \quad (51)$$

To solve this equation we start, for simplicity, by considering small σ . The equation (52) is reduced to

$$\left[-\partial_\sigma^2 - \frac{\lambda^2 w^2}{(1 - \lambda^2 E^2)^2 \sigma^4} \left(\frac{1 - \lambda E}{1 + \lambda E} \frac{N^2 - 1}{4} - \frac{\ell(\ell + 1)}{6} \right) \right] f_\ell^i = \frac{1 - \lambda E}{1 + \lambda E} w^2 f_\ell^i, \quad (52)$$

with the potential

$$V = \frac{-\lambda^2 w^2}{(1 - \lambda^2 E^2)^2 \sigma^4} \left(\frac{1 - \lambda E}{1 + \lambda E} \frac{N^2 - 1}{4} - \frac{\ell(\ell + 1)}{6} \right).$$

The potential V is quite zero for all E and only at $\sigma \approx 0.02$ that we see V varies in terms of E and goes up too fast to be close to zero as a constant function (Fig.4(a)).

The equation of motion (53) can be rewritten as follows

$$\left[-\frac{1+\lambda E}{1-\lambda E} \partial_\sigma^2 - \left((1+\lambda^2 \frac{N^2-1}{4(1-\lambda^2 E^2)^2 \sigma^4}) - \frac{1+\lambda E}{1-\lambda E} \frac{\lambda^2 \ell(\ell+1)}{6(1-\lambda^2 E^2)^2 \sigma^4} \right) w^2 \right] f_\ell^i = 0. \quad (53)$$

We change the coordinate to $\tilde{\sigma} = \sqrt{\frac{1-\lambda E}{1+\lambda E}} w \sigma$ and the equation (54) is rewritten as

$$\left[\partial_{\tilde{\sigma}}^2 + 1 + \frac{\kappa^2}{\tilde{\sigma}^4} \right] f_\ell^i(\tilde{\sigma}) = 0, \quad (54)$$

with

$$\kappa^2 = w^4 \lambda^2 \frac{3(1-\lambda E)^2(N^2-1) - 2(1-\lambda^2 E^2)\ell(\ell+1)}{12(1+\lambda E)^2(1-\lambda^2 E^2)^2}.$$

Then we follow the suggestions of WKB by making a coordinate change;

$$\beta(\tilde{\sigma}) = \int_{\sqrt{\kappa}}^{\tilde{\sigma}} dy \sqrt{1 + \frac{\kappa^2}{y^4}}, \quad (55)$$

and

$$f_\ell^i(\tilde{\sigma}) = (1 + \frac{\kappa^2}{\tilde{\sigma}^4})^{-\frac{1}{4}} \tilde{f}_\ell^i(\tilde{\sigma}). \quad (56)$$

Thus, the equation (55) becomes

$$(-\partial_\beta^2 + V(\beta)) \tilde{f}^i = 0, \quad (57)$$

with

$$V(\beta) = \frac{5\kappa^2}{(\tilde{\sigma}^2 + \frac{\kappa^2}{\tilde{\sigma}^2})^3}. \quad (58)$$

Then

$$f_\ell^i = (1 + \frac{\kappa^2}{\tilde{\sigma}^4})^{-\frac{1}{4}} e^{\pm i\beta(\tilde{\sigma})}. \quad (59)$$

Since we are dealing with small σ case the obtained fluctuation becomes

$$f_\ell^i = \frac{\tilde{\sigma}}{\sqrt{\kappa}} e^{\pm i\beta(\tilde{\sigma})}.$$

This is the asymptotic wave function in the regions $\beta \rightarrow -\infty$, while around $\beta \sim 0$; i.e. $\tilde{\sigma} \sim \sqrt{\kappa}$, $f_\ell^i \sim 2^{-\frac{1}{4}}$. The variation of this fluctuation in terms of small σ and the electric field is well shown in Fig.3(a) by considering the real part of the function. The variation of f_ℓ^i in terms of σ has positive values and goes up as σ goes up for all E in general. The influence of E on f_ℓ^i appears at $\sigma \approx 0.2$.

Now, if σ is too large the equation of motion (52) becomes

$$\left[-\partial_\sigma^2 + \frac{\ell(\ell+1)}{(1+\lambda E)\sigma^2} \right] f_\ell^i = \frac{1-\lambda E}{1+\lambda E} w^2 f_\ell^i. \quad (60)$$

The fluctuation solution of this equation is

$$\begin{aligned} f_\ell^i(\sigma) &= \alpha \sqrt{\sigma} \text{Bessel} J \left(\frac{1}{2} \sqrt{1 + 4 \frac{\ell(\ell+1)}{1+\lambda E}}, w \sigma \sqrt{\frac{1-\lambda E}{1+\lambda E}} \right) \\ &+ \beta \sqrt{\sigma} \text{Bessel} Y \left(\frac{1}{2} \sqrt{1 + 4 \frac{\ell(\ell+1)}{1+\lambda E}}, w \sigma \sqrt{\frac{1-\lambda E}{1+\lambda E}} \right), \end{aligned} \quad (61)$$

with α, β are constants. The variation of this fluctuation in terms of large σ and E is given by Fig.3(b). The values of f_ℓ^i are negative and they are going down as E going up.

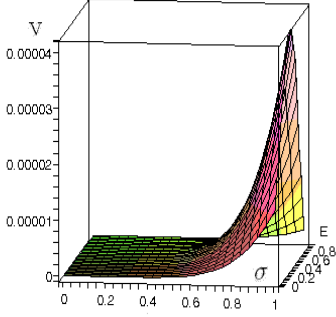


Figure1: V in zero mode of overall transverse fluctuations.

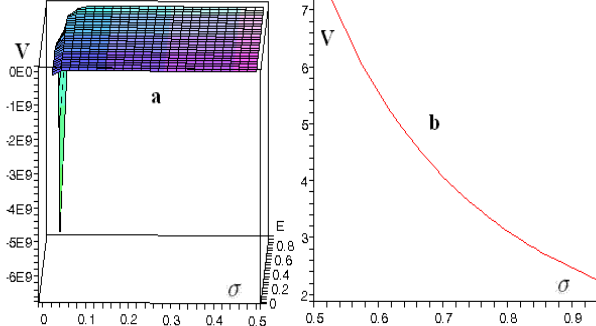


Figure 2 : V in non-zero mode of overall transverse fluctuations. (a): small σ and (b): large σ .

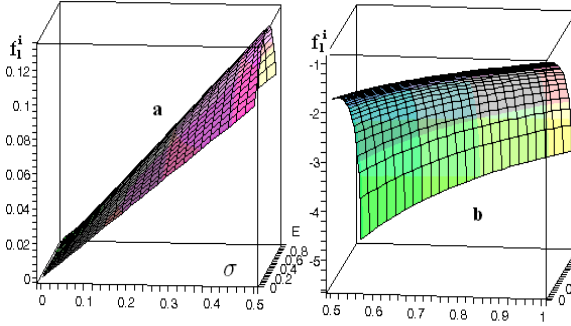


Figure 3: Relative transverse fluctuations in non-zero mode. (a) small σ with $\beta \sim 0$. (b) large σ .

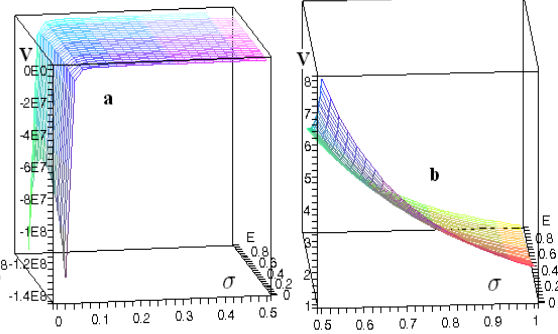


Figure 4 : V in non-zero mode of relative transverse fluctuations. (a): Small σ and (b): Large σ .

By dealing with the fluctuations (60) (Fig.3(a)) and (62) (Fig.3(b)) in small and large σ regions respectively, we remark that it is clear that we get different fluctuations from small to large σ with a singularity at some stage of σ and consequently the system is separated into two regions depending on the electric field.

The potential associated to (62) is

$$V(\sigma) = \frac{\ell(\ell+1)}{(1+\lambda E)\sigma^2}.$$

Accordingly to Fig.4(b) describing the variation of V , we remark that, V goes down as σ goes up and more down as E goes up; i.e. the potential becomes more small as the electric field appears between zero and $E \sim \frac{1}{\lambda}$. The potentials represented by the two figures Fig.4(a) and Fig.4(b) don't have an intersecting point at some stage of σ . This leads the system to get a singularity which supports the idea that the system is separated into two regions in non-zero modes of relative transverse fluctuations.

Consequently, the $D1 \perp D3$ system has Neumann boundary conditions and this is more clear at the presence of electric field. This is proved through this section by discussing different modes and different directions of the fluctuations of the funnel solution and their associated potentials. In the figures representing these variations we set w, ℓ equals to the unit of $\lambda = 1$ in all the treated equations and $N = 10$.

3.3 Overall Transverse Fluctuations in $D1 \perp D5$ System

We extend this study to discuss the electrified fluctuations in $D1 \perp D5$ system. We give the equations of motion of the fluctuations and their solutions. Then, we discuss the variation of the potential and the fluctuations in terms of electric field and the spatial coordinate.

We start by considering overall transverse fluctuations in zero mode and we let the non-zero modes with the relative transverse fluctuations to be discussed in the next project [22]. This type of fluctuations is given as $\delta\phi^m(\sigma, t) = f^m(\sigma, t)I_N$, $m = 6, 7, 8$. Plugging this fluctuation into the full (N, N_f) -string action (15'), together with the funnel (16) the action is found to be

$$S = -NT_1 \int d^2\sigma \left((1 + \lambda E)A - \frac{1}{2}(1 - \lambda^2 E^2)\lambda^2 A(\dot{f}^m)^2 + \frac{1}{2}(1 + \lambda E)\lambda^2 (\partial_\sigma f^m)^2 + \dots \right). \quad (62)$$

where

$$A = \left(1 + \frac{4R(\sigma)^4}{c\lambda^2}\right)^2, \quad (63)$$

with the quadratic terms in f^m were the only terms retained in the action. The linearized equation of motion of the fluctuation is then

$$\left[(1 - \lambda E)\left(1 + \frac{4R(\sigma)^4}{c\lambda^2}\right)^2 \partial_t^2 - \partial_\sigma^2\right] f^m = 0. \quad (64)$$

We consider small R which is given by (17) in the second section. We insert its expression in the last equation and the equation of motion becomes

$$\left[(1 - \lambda E)\left(1 + \frac{n^2\lambda^2}{16(1 - \lambda^2 E^2)^2\sigma^4}\right)^2 \partial_t^2 - \partial_\sigma^2\right] f^m = 0 \quad (65)$$

for large n .

Let's consider the fluctuation in the following form

$$f^m = \phi(\sigma)e^{-i\omega t}\delta x^m, \quad (66)$$

with δx^m , $m = 6, 7, 8$, the direction of the fluctuation. The equation (66) becomes

$$\left[-\partial_\sigma^2 - w^2(1 - \lambda E)\left(\frac{n^2\lambda^2}{8(1 - \lambda^2 E^2)^2\sigma^4} + \frac{n^4\lambda^4}{16^2(1 - \lambda^2 E^2)^4\sigma^8}\right)\right]\phi = w^2(1 - \lambda E)\phi. \quad (67)$$

The potential of this system is

$$V = -w^2(1 - \lambda E)\left(\frac{n^2\lambda^2}{8(1 - \lambda^2 E^2)^2\sigma^4} + \frac{n^4\lambda^4}{16^2(1 - \lambda^2 E^2)^4\sigma^8}\right) \quad (68)$$

depending on the electric field E with $E \in [0, \frac{1}{\lambda}[$.

3.3.1 Small σ Limit

The equation (68) is complicated and to simplify the calculations we start by considering the small σ and $\frac{1}{\sigma^8}$ dominates in (68) and (69). We then discuss the equation

$$\left[-\partial_\sigma^2 - w^2(1 - \lambda E)\frac{n^4\lambda^4}{16^2(1 - \lambda^2 E^2)^4\sigma^8}\right]\phi = w^2(1 - \lambda E)\phi, \quad (69)$$

and the potential is reduced to

$$V = -w^2(1 - \lambda E)\frac{n^4\lambda^4}{16^2(1 - \lambda^2 E^2)^4\sigma^8}. \quad (70)$$

As shown in Fig.5(a), the potential V tends to $-\infty$ until some values of E when E and σ are close to zero, and once E is close to the inverse of λ the potential is zero for all small σ . We consider in this case $\sigma \in]0, 0.5]$ in the unit of λ with $\lambda = 1$, $w = 1$, $n = 10$ and $E \in [0, 1[$.

To solve the differential equation (70), we consider the total differential on the fluctuation. Let's denote $\partial_\sigma \phi \equiv \phi'$. Since ϕ depends only on σ we find $\frac{d\phi}{d\sigma} = \partial_\sigma \phi$. We rewrite the equation (70) in this form

$$\frac{1}{\phi} \frac{d\phi'}{d\sigma} = -w^2(1 - \lambda E) \left[\frac{n^4 \lambda^4}{16^2(1 - \lambda^2 E^2)^4 \sigma^8} + 1 \right]. \quad (71)$$

An integral formula can be written as follows

$$\int_0^{\phi'} \frac{d\phi'}{\phi} = - \int_0^\sigma w^2(1 - \lambda E) \left[\frac{n^4 \lambda^4}{16^2(1 - \lambda^2 E^2)^4 \sigma^8} + 1 \right] d\sigma, \quad (72)$$

which gives

$$\frac{\phi'}{\phi} = -w^2(1 - \lambda E) \left[-\frac{n^4 \lambda^4}{16^2(1 - \lambda^2 E^2)^4 \times 7\sigma^7} + \sigma \right] + \alpha. \quad (73)$$

We integrate again the following

$$\int_0^\phi \frac{d\phi}{\phi} = - \int_0^\sigma (w^2(1 - \lambda E) \left[-\frac{n^4 \lambda^4}{16^2 \times 7(1 - \lambda^2 E^2)^4 \sigma^7} + \sigma \right] + \alpha) d\sigma. \quad (74)$$

We get

$$\ln \phi = -w^2(1 - \lambda E) \left[-\frac{n^4 \lambda^4}{16^2 \times 42(1 - \lambda^2 E^2)^4 \sigma^6} + \frac{2}{\sigma^2} \right] + \alpha\sigma + \beta, \quad (75)$$

and the fluctuation in small σ region is found to be

$$\phi(\sigma) = \beta e^{-w^2(1 - \lambda E) \left[-\frac{n^4 \lambda^4}{16^2 \times 42(1 - \lambda^2 E^2)^4 \sigma^6} + \frac{\sigma^2}{2} \right] + \alpha\sigma}, \quad (76)$$

with β and α are constants.

We plot the progress of the obtained fluctuation in (Fig.6(a)). First we consider the constants $\beta = 1 = \alpha$, then the small spatial coordinate in the interval $[0, 0.5]$ with the unit of $\lambda = 1$, $w = 1$ and $n = 4$. As above the electric field is in $[0, 1[$. We see that at the absence of the electric field there is no fluctuations at all and this phenomenon continues for the small values of E . When $E \approx 0.5$ the fluctuation appears from $\sigma = 0.15$ and goes down as σ and E go up.

3.3.2 Large σ Limit

In the large σ case, the equation (68) becomes

$$\left[-\partial_\sigma^2 - w^2(1 - \lambda E) \frac{n^2 \lambda^2}{8(1 - \lambda^2 E^2)^2 \sigma^4} \right] \phi = w^2(1 - \lambda E) \phi \quad (77)$$

and the potential is

$$V = -w^2(1 - \lambda E) \frac{n^2 \lambda^2}{8(1 - \lambda^2 E^2)^2 \sigma^4}. \quad (78)$$

By plotting the progress of this potential (Fig.5(b)) we consider the large spatial coordinate in the interval $[0.5, 1]$ and $E \in [0, 1[$ in the unit of $\lambda = 1$, $w = 1$ with $n = 10$. The obtained figure shows that V has in general higher values than the ones obtained in small σ case (Fig.5(a) describing (71)). Specially, for the first values of σ , V goes up from negative values to be close to zero for almost values of E until E is close to $\frac{1}{\lambda}$, approximately from $E = 0.8$ where $V \approx -0.02$, we remark that V has small variation in $[0.8, 1]$ region of σ . By contrary, in figure 5(a), when $\sigma = 0.5$ which is the last value of σ in that case we find V is already zero for all E . Consequently, these two potentials (71) and (79) show a big gap to go from one system to other that they describe, meaning that our system is separated into two regions; small and large σ depending on E .

Now, we should solve the equation of motion of the relative transverse fluctuations (78), in the case of large σ . We start by defining a new coordinate

$$\tilde{\sigma}^2 = w^2(1 - \lambda E)\sigma^2$$

and (78) becomes

$$\left(1 + \frac{n^2\lambda^2w^4(1 - \lambda E)^2}{8(1 - \lambda^2E^2)^2\tilde{\sigma}^4} + \partial_{\tilde{\sigma}}^2\right)\phi(\tilde{\sigma}) = 0, \quad (79)$$

with the potential is

$$V(\tilde{\sigma}) = \frac{\kappa^2}{\tilde{\sigma}^4}, \quad (80)$$

and

$$\kappa^2 = \frac{n^2\lambda^2w^4(1 - \lambda E)^2}{8(1 - \lambda^2E^2)^2}.$$

The equation (80) is a Schrödinger equation for an attractive singular potential $\propto \tilde{\sigma}^{-4}$ and depends on the single coupling parameter κ with constant positive Schrödinger energy. The solution is then known by making the following coordinate change

$$\chi(\tilde{\sigma}) = \int_{\sqrt{\kappa}}^{\tilde{\sigma}} dy \sqrt{1 + \frac{\kappa^2}{y^4}}, \quad (81)$$

and

$$\Phi = \left(1 + \frac{\kappa^2}{\tilde{\sigma}^4}\right)^{-\frac{1}{4}} \tilde{\Phi}. \quad (82)$$

Thus, the equation (80) becomes

$$\left(-\partial_{\chi}^2 + V(\chi)\right)\tilde{\Phi} = 0, \quad (83)$$

with

$$V(\chi) = \frac{5\kappa^2}{(\tilde{\sigma}^2 + \frac{\kappa^2}{\tilde{\sigma}^2})^3}. \quad (84)$$

Then, the fluctuation is found to be

$$\Phi = \left(1 + \frac{\kappa^2}{\tilde{\sigma}^4}\right)^{-\frac{1}{4}} e^{\pm i\chi(\tilde{\sigma})}. \quad (85)$$

This fluctuation has the following limit; since we are in large σ region $\Phi \sim e^{\pm i\chi(\tilde{\sigma})}$. This is the asymptotic wave function in the regions $\chi \rightarrow +\infty$, while around $\chi \sim 0$; i.e. $\tilde{\sigma} \sim \sqrt{\kappa}$,

$\Phi \sim 2^{-\frac{1}{4}}$. Owing to the plotting of the progress of this fluctuation given by Fig.6(b), by considering the real part of the function, we remark that Φ goes down fast as E goes up for all σ . When $\sigma = 0.5$ the fluctuation gets different values for all E compared to the values gotten in small σ region by (77) (Fig.6(a)). These two figures show that the fluctuations of fuzzy funnel of D1 \perp D5 branes have a singularity at some stage of σ separating the system into two regions; small and large σ .

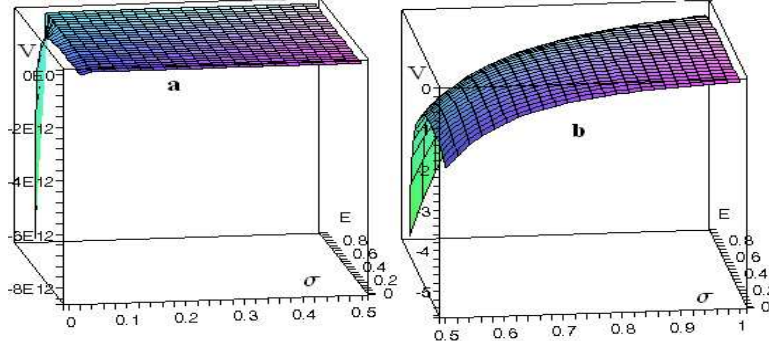


Figure 5: V in zero mode of overall transverse fluctuations. (a): small σ , (b): large σ .

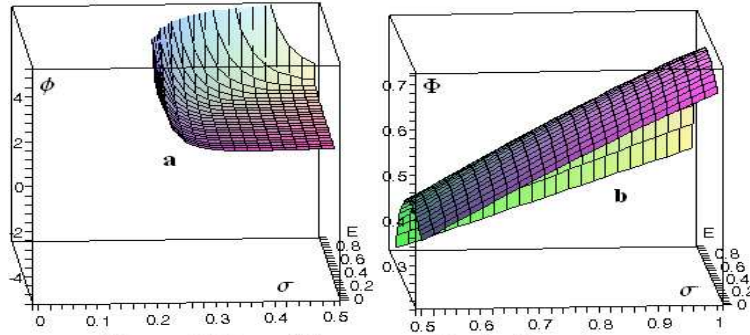


Figure 6: Overall transverse fluctuations in zero mode. (a): small σ , (b): large σ .

4 Discussion and Conclusion

In this work, we showed that certain excitations of D1 \perp D3 and D1 \perp D5 systems can be shown to obey Neumann boundary conditions. We considered the non-abelian BI dynamics of the dyonic string such that the electric field E has a limited value. The limit of E attains a maximum value

$$E_{max} = T_1 = \frac{1}{\lambda} \quad (86)$$

(for simplicity we dropped 2π in all the calculations). This limiting value arises because if $E > \frac{1}{\lambda}$ the action ceases to make physical sense [21]. The system becomes unstable. Since The string effectively carries electric charges of equal sign at each of its endpoints, as E increases the charges start to repel each other and stretch the string. For E larger than the critical value (87), the string tension T_1 can no longer hold the strings together.

In this context, by considering $E \in [1, \frac{1}{\lambda}[$ in D1 \perp D3 and D1 \perp D5 branes, we treated the fluctuations of the fuzzy funnel solutions and discussed the associated potentials V in terms of the electric field E and the spatial coordinate σ . We considered the unit of λ in all the figures representing the variations. We limited σ to be in the interval $[0, 0.5]$ for small σ and $[0.5, 1]$ for large σ .

Concerning $D1\perp D3$ system, we gave the variation of V in zero mode of overall transverse fluctuations in Fig.1; this figure shows that the system is looking like separated into two regions depending on the electric field. The potential is stable for E varies from 0 until 0.7 and then V goes up quickly as E close to $\frac{1}{\lambda}$. Then we dealt with the general case of the overall transverse fluctuations which is the non-zero modes. In this case, the idea that the system is divided into two regions appears more clear. We gave the figure representing the potential in Fig.2; we see that at $\sigma = 0.5$ the potential gets a singularity. We continued to treat the other kind of the fluctuations, it's the relative transverse fluctuations. These fluctuations are represented by Fig.3 and the associated potentials by Fig.4; we obtain the same remark that both fluctuations and potential have a singularity at $\sigma = 0.5$. This supported the idea that the system is separated into two regions.

We extended our study to the case of higher dimensions. We treated the electrified fluctuations of $D1\perp D5$ branes and we studied only the zero mode of overall transverse fluctuations. All other cases will be discussed in the coming work. We notice that when the electric field is going up and down the potential of the system is changing and the appearance of the singularity is more clear (Fig.5) and we have the same remarks for the fluctuations of fuzzy funnel solutions as well (Fig.6) which cause the division of the system into two regions depending on small and large σ and also on E .

Consequently, the end point of the dyonic strings moves on the brane which means we have Neumann boundary conditions in $D1\perp D3$ and $D1\perp D5$ branes. The physical interpretation is that a string attached to the $D3$ and $D5$ branes manifests itself as an electric charge, and the waves on the string cause the end point of the string to freely oscillate. Thus, we realize Polchinski's open string Neumann boundary conditions dynamically by considering non-abelian BI action in $D1\perp D3$ and $D1\perp D5$ systems.

In the coming work [22], we will focus on other interesting investigations concerning the perturbations propagating on a dyonic string in the supergravity background [19, 10] of an orthogonal p-brane. We will discuss the relative transverse fluctuations in dyonic $D1$ - $D5$ system in flat background and supergravity background.

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